



**COMPLEX FINANCIAL PRODUCTS:
PROPOSED EVALUATION METHODS
2010/01/14**

1. INTRODUCTION

During the last decade, the retail banking sector has increasingly offered sophisticated savings/investment products. Some of those products are competing directly with the traditional savings products, such as deposits that offer clients a constant return. With MiFID implementation, regulation and supervision of some of these financial products is at this time stricter. Complex financial products are included in this set.

Usually the return of these products depends on the performance of one or more underlying assets. The aggressiveness and risk of these new savings/investment products is quite diverse. A common problem that investors face is related to the evaluation of the returns of complex financial products. Thus, even investors who are quite skilled in financial market matters show some difficulty in understanding the peculiarities of the majority of these products.

In fact, the non-linear payoff structure of complex financial products is not easily understood by investors. That happens when payoffs exhibit high asymmetry and kurtosis (or even jumps), which in these cases means that the products link high return rates to certain unlikely scenarios. These unlikely scenarios are displayed to the public as a client-attraction factor. Notwithstanding this, the probability of success associated to those promoted scenarios is very low and the expected return of the products is usually lower than the return of traditional retail savings products.

This study analyses a number of complex financial products with the characteristics mentioned. Given the diversity of products available in the market with these features, it is not feasible to scrutinise them all in the present document. Despite this, a wide-ranging set of complex financial products was taken from the market, which included, inasmuch as possible, the main characteristics of this class of financial instruments.

The text has been divided into the following chapters: the second chapter handles theoretical issues which concern the quantitative models used for assessing and calculating probabilities of the various possible scenarios for complex financial products' returns; the third describes the various selected products and presents the results of that evaluation. A concluding summary is presented at the end.

2. QUANTITATIVE MODELS USED IN THE ANALYSIS

The 1970s were marked by an exponential growth in the derivatives markets in the US and in the main world financial market places. Scientific progress in the investigation of contingent financial asset pricing preceded that growth. The 1973 publication of the 'Pricing of Options and Corporate Liabilities' by Fischer Black and Myron Scholes marked a new era for the international financial markets, which gradually introduced innovative products based on the options theory.

The plain vanilla option pricing is usually derived from a closed-form formula: the Black-Scholes formula. However, the introduction of increasingly unconventional products in the market has led the financial industry to resort to alternative methods for assessing those products. Said methods include binomial (or trinomial) trees and Monte Carlo simulations (parametric and non-parametric).

The complex financial products analysed here largely depend on the evolution of two or more correlated underlying assets. Two alternative methods for assessing the expected average annual return (denoted as AAER) of complex financial products and their implied probabilities linked to several payoff payments are considered: the parametric Monte Carlo simulation and Bootstrapping.

2.1 PARAMETRIC MONTE CARLO SIMULATION

The Parametric Monte Carlo simulation¹ is used to model the payoffs of the underlying assets and can be applied to several stochastic processes. In this study, the following are considered: The Geometric Brownian Motion (GBM), the Variance Gamma Model (VGM), the Vasicek Model and the Cox- Ingersoll-Ross Model (CIR).

2.1.1 GEOMETRIC BROWNIAN MOTION

This stochastic movement was used in the Black and Scholes model² (1973). The model for contingent asset pricing proposed by the authors is based on a set of assumptions regarding the performance of the underlying asset prices, namely:

1. The prices of the underlying assets follow a Geometric Brownian Motion:

$$dS_{t+1}^i = \mu(t) \times S_t^i + \sigma(t) \times S_t^i \times dz$$

¹ The Monte Carlo method (MCM) is a statistical method for stochastic simulations and is used in several domains. The Monte Carlo method has, for some time now, been used hitherto to obtain numeric estimates of complex functions. This method typically involves generating observations for the estimation of the desired function. The most common uses of said method are the numeric calculation for assessing integrals. The method's idea is to use the integral that one intends to calculate as an expected value. Source: The Basics of Monte Carlo Simulation, University of Nebraska-Lincoln, Joy Woller, Spring 1996.

² Fischer Black and Myron Scholes, "The Pricing of Options and Corporate Liabilities", The Journal of Political Economy, Vol. 81, No. 3. (May - Jun., 1973), pp. 637 - 654.

where dz is a Wiener process, $\mu(t)$ denotes the expected yield rate at t and $\sigma(t)$ is the expected volatility at t .

The geometric movement used by Black and Scholes (1973) also assumes that $\mu(t)$ and $\sigma(t)$ are constant over time, which means that:

$$dS_{t+1}^i = \mu \times S_t^i + \sigma \times S_t^i \times dz$$

2. Short positions are allowed;
3. There are neither transaction costs nor taxes. All securities are perfectly divisible;
4. There are no arbitrage opportunities;
5. The securities are traded continuously;
6. The risk-free interest rate is identical for all maturities.

The Monte Carlo simulation is based on a risk neutral valuation. This method consists on the simulation of the path of payoffs in a risk neutral environment. The expected payoff is subsequently discounted at the risk-free interest rate.³

Assuming that the interest rate and the dividend yield for each security are constant, the path of each asset would be expressed as follows:

$$S_T^i = S_0^i \times \exp \left[\left(r - d - \frac{\sigma^2}{2} \right) \times dt + \sigma \times \epsilon \times \sqrt{dt} \right]$$

where ϵ is the random shock, and r and d are the risk-free interest rate and the dividend yield of the security, respectively.

Some complex financial products are structured in a way that the holders' remuneration depends on the performance of several assets whose returns may be strongly correlated. This means that the Wiener process random shocks of the several securities are also correlated, and should be adjusted in order to reflect that fact. Financial literature offers various methods to model correlation between financial securities, most notably, the Cholesky decomposition and the principal components method.⁴ The Cholesky decomposition seems to be the most appealing from a computational point of view and is thus used to incorporate the correlation between the various underlying assets used in simulations.

The ϵ variable may be obtained as follows:

- a) By generating k independent samples from a normal distribution (where k stands for the number of securities);
- b) By calculating the correlation matrix for several underlying assets;

³ The use of a risk-free interest rate in these models is due to the obligatory absence of arbitrage opportunities so that the pricing is fair.

⁴ Hull, J. (2008), *Options, Futures, and Other Derivatives*, 7th Edition Prentice Hall.

Les Clewlow and Strickland C.(1998), *Implementing Derivative Models*, Wiley Series in Financial Engineering).

- c) By applying the Cholesky decomposition to the correlation matrix of the underlying assets;
- d) Now, consider matrix $B_{(n \times k)}$ which features k independent samples (out of n observations) generated in a), and $J_{(k \times k)}$ is the matrix resulting from the Cholesky decomposition:

$$\epsilon_{(n \times k)} = B_{(n \times k)} \times J_{(k \times k)}$$

The $\epsilon_{(n \times k)}$ matrix provides the random shock for k underlying assets that influence the payoffs of the complex financial product. Column k of this matrix will contain n simulations for the price of underlying asset k .

After obtaining n simulations of the path followed by each of the k assets, it will be possible to obtain the probabilities of each scenario, in a risk neutral environment.

Cholesky Decomposition

A Monte Carlo simulation for returns of assets which are correlated requires generating random numbers from a multivariate normal distribution function. A method that is frequently used for generating numbers is based on the Cholesky decomposition. The following steps may be used for generating random numbers:

- 1 - Generation of k independent samples from a normal distribution (generated using the Microsoft Excel algorithm) – $x(k)$
- 2 - Calculation of the correlation between the k series of the underlying asset returns - ρ_{ij}
- 3 – Obtaining k dependent samples from the Cholesky decomposition's matrix (ϵ_k).

$$\epsilon_1 = \alpha_{11}x_1$$

$$\epsilon_2 = \alpha_{21}x_1 + \alpha_{22}x_2$$

$$\epsilon_3 = \alpha_{31}x_1 + \alpha_{32}x_2 + \alpha_{33}x_3 \dots$$

The α_{ij} coefficients are obtained with the Cholesky decomposition. For three underlying assets, the coefficients are calculated as follows:

$$\alpha_{11} = 1; \quad \alpha_{21} \Rightarrow \alpha_{21}\alpha_{11} = \rho_{21}$$

$$\alpha_{22} \Rightarrow \alpha_{21}^2 + \alpha_{22}^2 = 1$$

$$\alpha_{31} \Rightarrow \alpha_{31}\alpha_{11} = \rho_{31};$$

$$\alpha_{32} \Rightarrow \alpha_{31}\alpha_{21} + \alpha_{32}\alpha_{22} = \rho_{32}$$

$$\alpha_{33} \Rightarrow \alpha_{31}^2 + \alpha_{32}^2 + \alpha_{33}^2 = 1$$

Generalising

$$\alpha_{n1} = 1; \quad \alpha_{n1}\alpha_{11} = \rho_{n1}$$

...

$$\alpha_{n\ n-1} \Rightarrow \alpha_{n1}\alpha_{n-1\ 1} + \alpha_{n\ n-1}\alpha_{n-1\ n-1} = \rho_{n\ n-1}$$

$$\alpha_{nn} \Rightarrow \alpha_{n1}^2 + \alpha_{n2}^2 + \dots + \alpha_{nn}^2 = 1$$

2.1.2 VARIANCE GAMMA PROCESS

Despite its popularity, the Black-Scholes model has its limitations. On the one hand, some of the model's assumptions are unrealistic. On the other hand, empirical evidence shows that traders and investors use 'valuation models' which differ from the Black-Scholes model. A trader wanting to make the best possible decisions should not disregard the limitations of theoretical models. Consequently, despite the importance of the Black-Scholes option pricing model, investors seek ways to reduce potential errors due to the model's drawbacks. Said evidence was shown by the «volatility smiles» study.

Some researchers showed that the implied volatility of plain vanilla options was a function of the strike price. A possible explanation for volatility smiles lies in the perception of investors regarding potential errors due to the use of the log-normal distribution function to model the path of the underlying asset price. The high kurtosis and the skewness in the distribution function of the underlying asset's returns were explanations for volatility smiles.⁵ A high kurtosis means that the likelihood of an event occurring near to the average is higher than in the case of the normal distribution.

Madan, Carr and Chang (1998)⁶ expand an alternative valuation model based on the Variance-Gamma process. That model is a jump diffusion model which allow us to accommodate return distributions (of underlying assets) with kurtosis higher than 3. The model uses the following equations:

$$w = \frac{T}{v} \times \ln \left(1 + \theta \times v - \frac{\sigma^2}{2} \times v \right)$$

$$g = \text{GammaInv} \left(\phi(\epsilon), \frac{T}{v}, v \right)$$

$$\lambda = g \times \theta + \sqrt{g} \times \sigma \times \phi(\epsilon)$$

$$S_T^i = S_0^i \times \exp[(r - d) \times T + \lambda + w]$$

where v and θ denote the kurtosis and skewness of the distribution function of the underlying asset's return, σ is the underlying asset's volatility, d the dividend yield, r the risk-free interest rate, S_t^i the underlying asset price at t , T the time-to-maturity and ϵ the random shock. The Cholesky decomposition is also used to model the return's correlation structure.

⁵ This method was applied to the products with higher than normal kurtosis and skewness.

⁶ Madan, Carr and Chang. "The Variance-Gamma process and option pricing". European Finance Review 2 (1998).

2.1.3 VASICEK MODEL

The aforementioned stochastic processes are based on the assumption that the underlying asset follows a drift. Nonetheless, a specific class of underlying assets do not follow that trend and conversely revert to a long-run equilibrium value. This class of variables includes interest rates, inflation rates and exchange rates. The stochastic process used in the Vasicek model is as follows:

$$dr_{t+1}^i = a(b - r_t^i) \times dt + \sigma \times dz$$

where r_t^i is the reference value (interest rate, inflation rate, etc), a (>0) is the mean speed of reversion, b is the long-term mean value of the variable, σ is the volatility and dz is a Wiener process.

The mean reversion process occurs as follows: when r_t^i is higher (lower) than the long-run equilibrium value (b), the latter is force-pulled back to its equilibrium value. Said force will depend on a , i.e., the greater this parameter, the faster the dependent variable will converge towards the equilibrium value.

However, the Vasicek model has an undesirable feature: it allows for a negative dependent variable. This is hardly appealing for simulating the evolution of interest rates or even exchange rates.

2.1.4 COX-INGERSOLL-ROSS MODEL (CIR)

In the Cox-Ingersoll-Ross model, the path of the dependent variable also shows a mean reversion. Nonetheless, this stochastic process implies that the dependent variable shall not assume negative values, being thus especially attractive for interest rate simulations.

$$dr_{t+1}^i = a(b - r_t^i) \times dt + \sigma \times \sqrt{r_t^i} \times dz$$

The meaning of the aforementioned variables is identical to the variables used in the Vasicek model.

2.2 BOOTSTRAPPED HISTORICAL SIMULATION

One of the major criticisms to the previous approach is the assumption that the underlying assets' returns follow a parametric distribution. An alternative approach is the bootstrapped historical simulation which uses the empirical distribution of the underlying assets' returns. The advantage of bootstrapping over analytical methods is threefold: its simplicity; it does not require returns to have a parametric

distribution; and it does not require any correlation or standard deviation calculations of returns.

Bootstrapping was introduced by Efron in the late 70s.⁷ It is a non-parametric technique that may at times replace the theoretical statistical analysis with computation, which is increasingly available and becoming less expensive. The bootstrapping idea is that the original sample represents the population from which it was drawn. Resamples from this sample represent what we would get if we would take many samples from the population. The sampling is carried out with replacement.

Consider the following data sample:

$$x = \{x_1, x_2, \dots, x_n\}$$

- i. From x , k samples (with replacement) are generated;
- ii. From the k generated samples, the number of favourable cases are calculated, and thus it is possible to obtain the probability of each possible scenario.

2.3 SIMULATION ASSUMPTIONS

Common assumptions were made in all the simulations used for the evaluation of the products. 25.000 simulations for the Monte Carlo method (parametric models) and 1.000 simulations for the bootstrapping method were performed.

The proxy used for the risk-free rate was the 6-month Euribor interest rate, and the dividend yield was obtained from the last 5-year average of the underlying asset. The expected volatility was obtained from the underlying asset historical volatility of the 180 days prior to the product's issuance. The correlation structure between the securities was obtained from the historical data and the Cholesky decomposition was used.

3. COMPLEX FINANCIAL PRODUCTS - EVALUATION

3.1 CFP A

a) Product Description

Complex Financial Product (CFP) A is a capital guaranteed product, its return depending on the performance of 5 energy sector company shares. The product provides the half-yearly payment of a 6% coupon (GANR – Gross Annual Nominal Interest Rate) during a three-year period. However, said payment will depend on

⁷ Bradley Efron, "Bootstrap Methods: Another Look at the Jackknife", *Ann. Statist. Volume 7, N° 1 (1979)*, pp. 1-26.

the price path of Share 1, Share 2, Share 3, Share 4 and Share 5, as described below.

Consider S_t^i the share i spot price on date t (S_0^i is the share price at the moment the product is issued), X^i a lower barrier for share i below which no coupon payment will occur and B^i the upper barrier in excess of which early reimbursement (knock out) of the product occurs.

Observation Date (t)
26-04-2010
27-10-2010
26-04-2011
27-10-2011
26-04-2012
29-10-2012

The lower barrier for share i (X^i) corresponds to 85% of the reference price of share i on the date of the issuance, while B^i equals the reference price.

$$\begin{cases} X^i = 0.85 \times S_0^i \\ B^i = S_0^i, \end{cases} \quad \text{and } i = 1, \dots, 5$$

Thus, if at t :

- a) $S_t^i \geq 0.85 \times S_0^i, \forall i = 1, \dots, 5$, coupon payment will take place. This means that there will be no coupon payment if any of the prices of the aforementioned shares fall below 85% the initial reference price;
- b) $S_t^i \geq S_0^i, \forall i = 1, \dots, 5$, then early reimbursement of the invested capital and relevant coupon payment will take place.

b) Product Evaluation

The assessment of this product was conducted using the Monte Carlo simulation from the Geometric Brownian Motion stochastic process and bootstrapping. The probability of coupon payment is 25.4% and 27.6% at the end of the first half-year for the parametric model and the historical simulation, respectively.

The probability of the coupon payment decreases significantly over time partially due to the possibility of a knock-out barrier (should the prices of all shares be equal to or greater than 100% of the reference value).

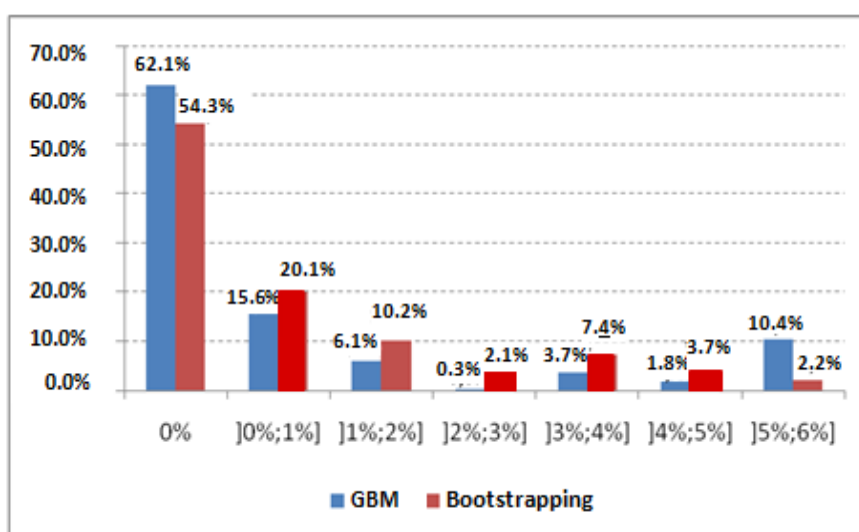
Table 1 – Coupon Payment Probability - CFP A

	GBM			Bootstrapping		
	0%	3%	Knock-out	0%	3%	Knock-out
26-04-2010	74.6%	25.4%	0.0%	72.4%	27.6%	0.0%
27-10-2010	80.5%	12.2%	7.3%	78.0%	18.0%	4.0%
26-04-2011	81.9%	7.4%	10.8%	79.5%	13.5%	7.0%
27-10-2011	82.5%	4.6%	12.9%	82.5%	7.8%	9.7%
26-04-2012	82.4%	3.3%	14.3%	82.7%	6.0%	11.3%
27-10-2012	82.4%	2.4%	15.2%	83.4%	3.8%	12.8%

The expected AAER (expected average annual return) obtained from the first method is 1.10%. Most simulation outcomes meant no payment for the investor (62.1% of the cases), meaning that the expected probability of a nil return during the product's entire lifespan is 62.1%. In 21.7% of the simulations, the AAER is between 0% and 2%. The 6% coupon payment over the entire three year period exists in 0.1% of the simulations.

The historical simulation results (bootstrap) show an expected AAER of 0.96% and a 54.3% probability of non-payment of any coupon during the contract's lifespan. In 30.3% of the observations, a 0% to 2% AAER is paid. There is a 2.2% probability of a 6% coupon in all the reference dates until maturity.

Chart 1 – AAER Simulations – CFP A



The probability of early reimbursement of invested capital (knock-out barrier) is analysed in the table that follows. The probability of said early reimbursement at the first-half-year is higher if the parametric simulation is used (7.3%).

	GBM	Bootstrapping
1st Half-Year	7.3%	4.0%
2nd Half-Year	3.5%	3.0%
3rd Half-Year	2.2%	2.7%
4th Half-Year	1.3%	1.6%
5th Half-Year	0.9%	1.5%
6th Half-Year	84.8%	87.2%

A last note concerns the fact that the Variance-Gamma Model has been excluded from the assessment of this product. In effect, some of the underlying assets show considerably high kurtosis (greater than 10 – see annex), which may result in the inappropriateness of this model.

3.2 CFP B

a) Product Description

CFP B is a debt instrument with capital guarantee. Its return depends on the performance of 5 mining sector company shares. There is either a 1% annual coupon or a 5% annual coupon, during the 5-year period. Payment will depend on the performance of the price of the following company shares: Share 6, Share 7, Share 8, Share 9 and Share 10.

Consider S_t^i the share i spot price on date t , S_0^i the share price at the time of the product's issuance and β_t^i the barrier at date t .

Year	Barrier (β)	Observation Date	Interest Payment Date
1	100%	6 October 2010	21 October 2010
2	95%	6 October 2011	21 October 2011
3	90%	6 October 2012	21 October 2012
4	85%	6 October 2013	21 October 2013
5	80%	6 October 2014	21 October 2014

At t , coupon payment will take place at:

- a) 5%, if the price of all the 5 shares is equal to or greater than the initial price multiplied by the barrier, i.e. if $S_t^i \geq \beta_t \times S_0^i, \forall i = 1, \dots, 5$;
- b) 1%, if the price of at least one of the shares is less than the initial price multiplied by the barrier, i.e. if $\exists i | S_t^i < \beta_t \times S_0^i, i = 1, \dots, 5$.

b) Product Evaluation

The GBM method shows no meaningful differences in the 5% coupon payment probability (varying from a maximum value of 11% in the last year to a minimum of 8.7% in the third year). However, using the bootstrap method, a significant increase in the payment probability emerges during those 5 years, ending up with a 29.1% probability in the fifth year. The 1% coupon payment probability is circa 89% to 91.3% or 70.9% to 86.3%, with the GBM and bootstrap, respectively.

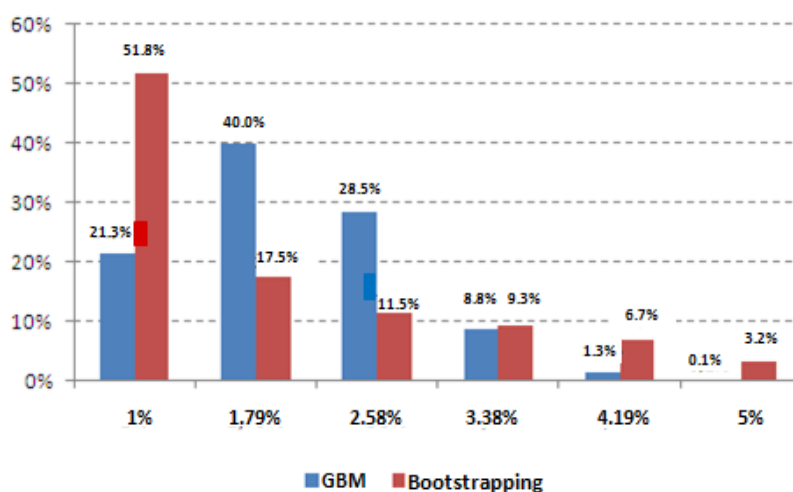
Table 2 – Coupon Payment Probability – CFP B

Coupon	GBM		Bootstrap	
	1%	5%	1%	5%
1st year	90.1%	9.9%	86.3%	13.7%
2nd year	90.6%	9.4%	81.5%	18.5%
3rd year	91.3%	8.7%	76.2%	23.8%
4th year	90.3%	9.7%	73.9%	26.1%
5th year	89.0%	11.0%	70.9%	29.1%

Chart 2 shows the frequency distribution for this product's AAER. Using the Monte Carlo method as from the Geometric Brownian Motion stochastic process, one can see that in 61.3% of the cases, this product has a AAER of less than 2%, and a 5% AAER in 0.1% of the cases. The expected gross nominal annual rate is 1.39% at maturity for the whole 5-year period.

Using the bootstrap method, the AAER for the 5-year period is 1.89%. In circa 51.8% of the simulations, a 1% coupon payment occurs and in only 3.2% is the 5%/year maximum coupon paid, although this percentage is higher than the one obtained via the GBM method.

Chart 2 – AAER Simulations – CFP B



3.3 CFP C

a) Product Description

This is a capital guaranteed product which return depends on the stock market price path of the underlying assets (three indices – Index 1, Index 2 and Index 3). It has a 5-year maturity, and, depending on the behaviour of the underlying assets' prices, it may or may not pay a 6.6% annual coupon. The observation dates of the indices' performance and the coupon payment dates are as follows:

Observation Date (t)	Payment Date (t1)
23-11-2010	30-11-2010
23-11-2011	30-11-2011
23-11-2012	30-11-2012
25-11-2013	02-12-2013
24-11-2014	01-12-2014

The 6.6% coupon payment will depend on the price path of the indices, which are a set of three commodity indices, and on the evolution of the Consumer Price Index (CPI) disclosed by Eurostat for the euro area (Eurostat Consumer Price Index (excluding Tabaco) for the euro area).

The three indices form an equal weight basket. Consider I_t the value of the basket on date t and IPC_t the value of the Consumer Price Index disclosed by Eurostat for the euro area. Two possible scenarios are available for each of the observation dates (t):

Scenario 1

$$\frac{I_t}{I_0} - 1 \geq \frac{IPC_t}{IPC_0} - 1 \quad \rightarrow 6.6\% \text{ Coupon Payment}$$

Scenario 2

$$\frac{I_t}{I_0} - 1 < \frac{IPC_t}{IPC_0} - 1 \quad \rightarrow \text{There is no coupon payment}$$

Thus, at each observation date, the average value of the basket is assessed against the change in the Consumer Price Index. If the change of the basket's value is greater than the change of the CPI index, the 6.6% coupon will be paid. If not, no coupon will be paid.

b) Product Evaluation

The return on this product depends on the performance of the three commodity indices, which in turn depend on the path of the underlying commodity prices, and on the path of the Consumer Price Index. The analysis was divided in two stages: in the first stage, the commodity indices path was simulated; in the second stage, the inflation rate path linked to the Consumer Price Index was simulated.

Two different models were used for the evolution of the commodity indices' prices: the Monte Carlo simulation from the Geometric Brownian Motion stochastic process and the bootstrap. Two different methods were used for the Consumer Price Index's evolution: the parametric simulation (Vasicek model) and the historical simulation (bootstrap).

The probability of the coupon payment is 44.8% at the end of the first year, should one consider the parametric model. The probability of the coupon payment drops slightly during subsequent years, reaching 39.5% by the end of the 5th year. Using the historical simulation, there is a slight increase in the probability of coupon payment by the end of the second year (43.0% probability) and a slight drop at the end of the subsequent periods. This means that the probability of non-payment of any coupon is always greater than 55%, regardless of the method used.

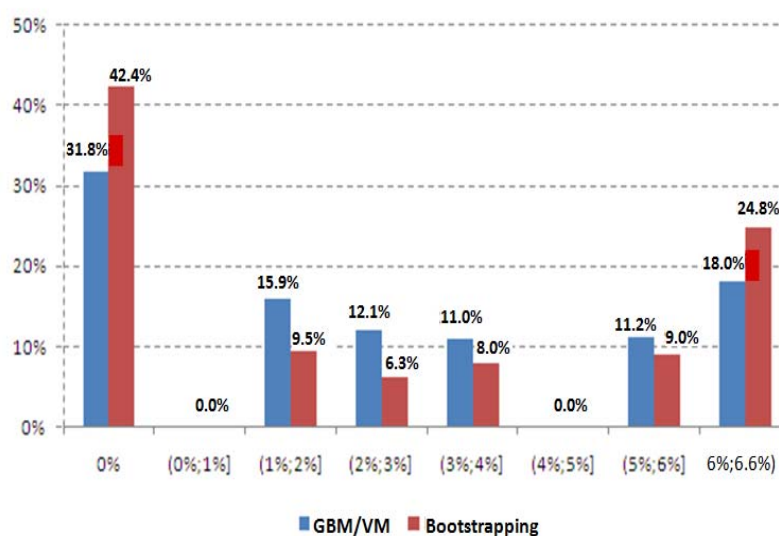
Table 3 – Coupon Payment Probability – CFP C

Coupon	GBM/VM		Bootstrapping	
	0%	6.6%	0%	6.6%
Year 1	55.2%	44.8%	57.4%	42.6%
Year 2	57.4%	42.6%	57.0%	43.0%
Year 3	58.9%	41.1%	58.2%	41.8%
Year 4	60.0%	40.0%	60.2%	39.8%
Year 5	60.5%	39.5%	61.1%	38.9%

The results of the parametric simulations show a 2.72% AAER. The product has guaranteed capital and there is a 31.8% probability that its return will be nil. On the other hand, the probability of coupon payment on the five observation dates reaches 18.0%.

Using the non-parametric simulations, the AAER for the product is 2.71%. This value is close to that obtained in the parametric simulation. However, the payment distribution in the two simulations diverges. Thus, the probability of non-payment of any coupon increases from 31.8% to 42.4%. On the other hand, the coupon payment probability during the five years increases from 18.0% to 24.8%.

Chart 3 – AAER Simulations – CFP C



A last note concerns the fact that the Variance-Gamma Model has not been used. The use of the stochastic process linked to this model implies that the series of returns of the underlying assets should have a kurtosis greater than 3 (i.e. greater than the normal distribution kurtosis). However, from the empirical distribution functions of the returns of the three commodity indices we conclude that only one series fulfilled that condition, rendering this model unsuitable for the analysis envisaged.

3.4 CFP D

a) Product Description

CFP D consists on 3-year maturity bonds that are subject to early redemption. Its return is linked to the three share indices' evolution: Index 4, Index 5 and Index 6. The product has the following characteristics:

There are two possible scenarios at the end of the first year:

- If the official closing value on observation date 1 for all indices is equal to or greater than 100% of the initial value (observed on the issuance date), there will be an automatic early redemption of the bonds at 100% of the invested capital plus the gross annual coupon of 5%;
- If not, there will be no coupon payment and the bonds will be carried over to the following year.

If there is no early redemption by the end of the second year, two scenarios are possible:

- If the official closing value on observation date 2 for all indices is equal to or greater than 105% of the initial value (observed on the issuance date), there will be an automatic early redemption of the bonds at 100% of the invested capital plus the gross annual coupon of 10%;
- If not, there will be no coupon payment and the bonds will continue invested until maturity date.

At the end of the third year (maturity date), if no early redemption took place before, two scenarios are possible:

- If the official closing value on the date of observation for all three indices is equal to or greater than 110% of its initial value (observed on the issuance date), there will be repayment of the bonds at 100% of the invested capital plus a gross annual coupon of 15%;
- Otherwise, there will be no coupon payment and 100% of the invested capital shall be repaid.

<i>i</i>	<i>Observation Date i</i>	<i>Early Redemption Date and Maturity Date</i>
1	23 October 2010	30 October 2010
2	23 October 2011	30 October 2011
3	23 October 2012	30 October 2012

b) Product Evaluation

It is necessary to simulate the path of the three share indices. The models used are the Geometric Brownian Motion (GBM), the Variance Gamma Model (VGM) and the Bootstrap model.

The probability of non-payment for any coupon ranges between 49.9% (GBM) and 89.0% (bootstrap). On the other hand, the probability of coupon payment is approximately 33.5% and 32.6% at the end of the first year, when taking the GBM and VGM models into consideration, and only 8% for the bootstrap model. The probability of coupon payment decreases substantially in the second and third years.

Table 4 – Coupon Payment Probability – CFP D

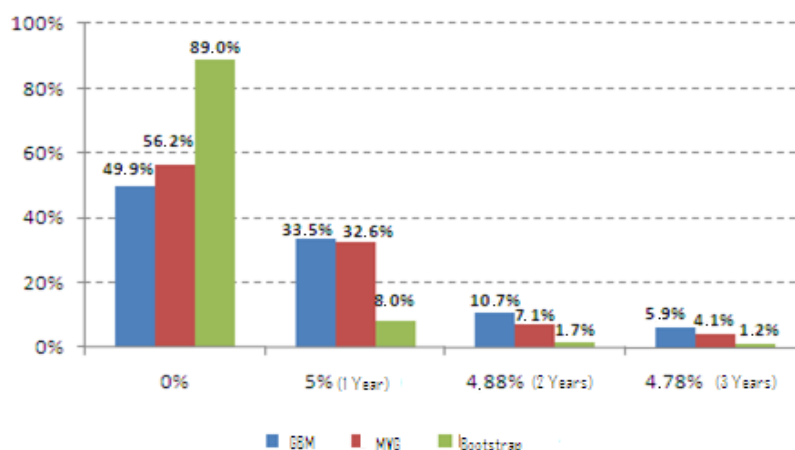
	<i>GBM</i>	<i>VGM</i>	<i>Bootstrap</i>
0%	49.9%	56.2%	89.0%
5% Coupon --> Year 1	33.5%	32.6%	8.0%
10% Coupon --> Year 2	10.7%	7.1%	1.7%
15% Coupon --> Year 3	5.9%	4.1%	1.2%

According to the Monte Carlo method from the stochastic process of the Geometric Brownian Motion, the probability of a 5% coupon payment in the first year is 33.5%. These bonds are thus redeemed early and are not carried over to the next observation date. In the second year, only 10.7% of the simulations lead to the payment of a 10% gross annual coupon (4.88% average annual expected return). Only 5.9% of the simulations lead to the payment of the possible maximum gross annual coupon of 15% (4.78% average annual expected return). Around about half the simulations refund only the invested capital (0% interest). The average annual expected return obtained by this method is 2.48%.

The analysis of the frequencies linked with the use of VGM shows that in 56.2% of the cases the investor's return (average annual expected return) is 0%. Furthermore, 32.6% of investors experience a knock out in the first year with an average annual expected return of 5%, and only 4.1% reach the product's maturity with an average annual expected return of 4.78%. There are substantial differences between the results obtained by this method and those obtained from the Geometric Brownian Motion.

Bootstrap model results point to an average annual expected return of 0.54%, i.e. significantly lower than the average annual expected return obtained from Monte Carlo simulations. The probability linked to the non-payment of return is 89.0% and the probability of a 5% coupon payment in the first year is only 8.0%.

Chart 4 – AAER Simulations – CFP D



3.5. CFP E

a) Product Description

This is a product where neither return nor capital are guaranteed. Ultimately, the subscriber may lose the entire capital. The maturity is 5 years but may be subject to early repayment.

The return of this complex financial product will depend on three different assets (shares): Share 11, Share 12 and Share 13. The dates for any early repayment and final maturity are as follows:

<i>Observation Date</i>
<i>(t)</i>
21-10-2010
21-10-2011
21-10-2012
21-10-2013
21-10-2014

Early repayment is carried out in specific circumstances. If the individual valuation of all underlying assets is greater than or equal to 0, there will be early repayment of the invested capital plus a 42.5% coupon. Mathematically, the structure of payments on the dates of observation of a possible early redemption will be expressed as follows:

$$\left\{ \begin{array}{l} MR_t = 142.5\% \times VN \quad \text{if } \min_{i=1}^3 \left[\frac{\text{Underlying Asset Price } \frac{i}{t}}{\text{Underlying Asset Price } \frac{i}{0}} - 1 \right] \geq 0\%, t = 1, 2 \dots 4 \\ \\ MR_t = 0 \quad \text{if } \min_{i=1}^3 \left[\frac{\text{Underlying Asset Price } \frac{i}{t}}{\text{Underlying Asset Price } \frac{i}{0}} - 1 \right] < 0\%, t = 1, 2 \dots 4 \end{array} \right.$$

where MR_t is the amount to be repaid at t and VN is the nominal value subscribed.

In the event of no early repayment occurring between 2010 and 2013, the product's maturity will fall due on 21/10/2014. The payoff to be received by the product's subscriber will depend on the individual valuation of the invested capital:

$$1) \quad \min_{i=1}^3 \left[\frac{\text{Underlying Asset Price } \frac{i}{5}}{\text{Underlying Asset Price } \frac{i}{0}} - 1 \right] \geq -25\% \leftrightarrow MR = 142.5\% \times MN$$

$$2) \quad \min_{i=1}^3 \left[\frac{\text{Underlying Asset Price } \frac{i}{5}}{\text{Underlying Asset Price } \frac{i}{0}} - 1 \right] < -25\%$$

$$\leftrightarrow MR = \left(42.5\% + \min_{i=1}^3 \left[\frac{\text{Underlying Asset Price } \frac{i}{5}}{\text{Underlying Asset Price } \frac{i}{0}} \right] \times \Delta FX \right) \times MN$$

where, MR_t corresponds to the amount to be repaid at t , VN is the nominal value subscribed and ΔFX concerns the variation of the currency which is quoted in the underlying asset and the Euro, between 21/10/2009 and 21/10/2014.

This means that if no underlying asset drops more than 25% between the product's issuance and maturity, the nominal value of the product plus a 42.5% coupon will be paid. In the event of one (or more) underlying asset tumbling more than 25% compared with the initial value, the investor will receive the product's nominal value minus the largest depreciation of the three underlying assets at maturity (also taking into account the negative change in the currency in which the underlying asset is expressed) together with a 42.5% coupon.

b) Product Evaluation

The valuation of this product was conducted in two phases. In the first phase, the path of three shares was simulated. In the second phase, a simulation of the EUR/USD exchange rate was undertaken.

In order to simulate the trend of the three securities' pricing, three different models were used: the Geometric Brownian Motion (GBM), the Variance Gamma Model (MVG) and the bootstrap model. The same techniques were used to simulate the EUR/USD exchange rate.

From this product E onwards, the probability of interim coupon payments or other returns shall not be shown herein, since both for this or other products not yet analysed, the payment distribution is continuous (not binary as has been the case so far).

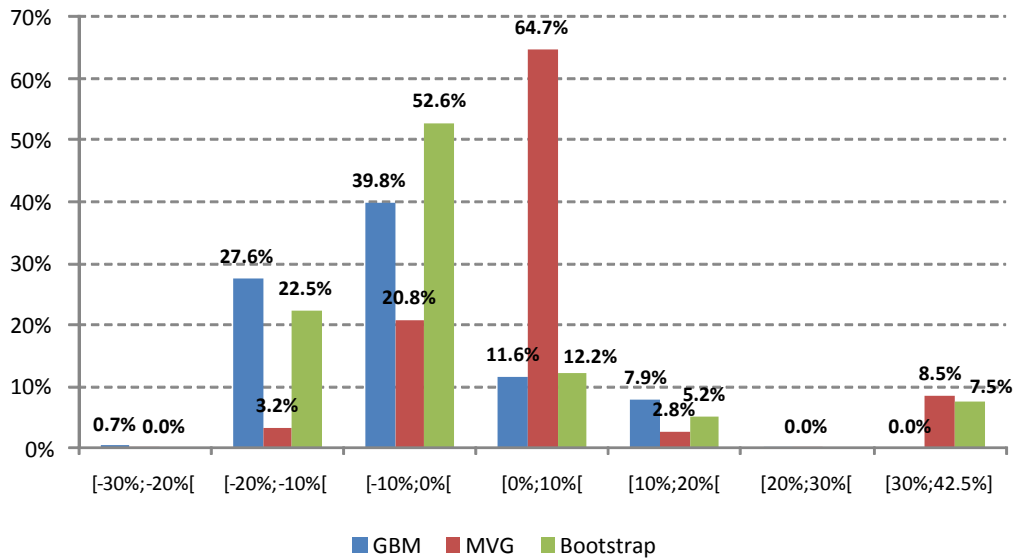
The average annual expected returns are 1.03%, 5.90% and -1.05%, following from the GBM, MVG and the bootstrap model, respectively.

Chart 5 shows the frequencies of the average annual expected return in the intervals found between the three methods used. It is concluded that for the simulation obtained from the Geometric Brownian Motion, in 39.8% of the cases, the complex financial product entails a loss between -10% and 0% on the capital invested. In 27.6% of the cases, the loss will lie between -20% and -10%. The probability of a positive return for investors is 31.9%, against the likelihood of loss of capital invested, which reaches 68.1%.

The MVG frequency table shows a greater concentration of returns (average annual expected return) for the investor in the 0% and 10% interval (64.7% of observations are within this interval). The interval with the second highest probability (20.8% of cases) is the [-10%, 0%[interval. These results are linked to strong negative skew and high kurtosis shown by the distribution function of returns for the securities' AAER under analysis.

This stochastic process accommodates the high kurtosis displayed by the series of returns. As the critical point for the coupon payment is close to the distributions' median, a critical mass of probability close to the reference quotation is reflected in a sharp upsurge in the probability of coupon payment. Thus, the combination of negative skew with the kurtosis shown by the series of returns contributes to a greater concentration of observations in the [-10%, 10%[interval. The overall results for this methodology mean that the loss probability of part of the invested capital is 68.1%, 25.0% and 75.1% respectively for the GBM, MVG and bootstrap methods.

Chart 5 – AAER Simulations – CFP E



The third method was the bootstrap historical simulation. Not modelling the products' correlation structure may impact its value. Thus, as the 42.5% coupon is payable in advance if the price of the three securities is above 'par', high positive correlations should result in a greater probability of earnings. If this correlation is neglected, the value of the complex financial product may be underestimated. Nevertheless, a historical simulation for the securities in question and for the exchange rate was conducted. The probability of obtaining a positive return is substantially reduced when compared to the parametric simulations' results. The most likely scenario is obtaining a return (Average Annual Expected Return) between -10% and 0% (52.6% probability).

Another important feature of this product is the possibility of early repayment (see Table below). The probability of repayment (which may be partial) only at maturity is 78.5%, 90.3% and 86.4% for the Geometric Brownian Motion, Variance Gamma Model and Bootstrap model, respectively. Out of the simulations carried out under the stochastic process of GBM, repayment would take place in the 4th year in 21.5% of the cases. In other years, the probability of early repayment would be nil. The MVG and bootstrap models indicate higher probabilities of early repayment in the first year (8.5% and 7.5% respectively).

<i>Redemption (Year)</i>	<i>GBM</i>	<i>VGM</i>	<i>Bootstrap</i>
1	0.0%	8.5%	7.5%
2	0.0%	1.0%	3.3%
3	0.0%	0.0%	1.9%
4	21.5%	0.2%	0.9%
5	78.5%	90.3%	86.4%

3.6. CFP F

a) Product Description

This product guarantees the capital at maturity. The maturity of the product is five years, with no interim coupon payment. The underlying asset is Index 7. The return of the product at maturity will match the percentage difference between the quarterly average price of the underlying asset until maturity and its initial reference value, shall this reference be positive, and 0 shall it be negative.

$$\text{Max} \left\{ \frac{1}{I_0} \text{Average}_{t=1}^T \{I_t\} - 1; 0 \right\}$$

where I_t is the underlying assets' value in quarter t and I_0 is the initial reference value.

b) Product Evaluation

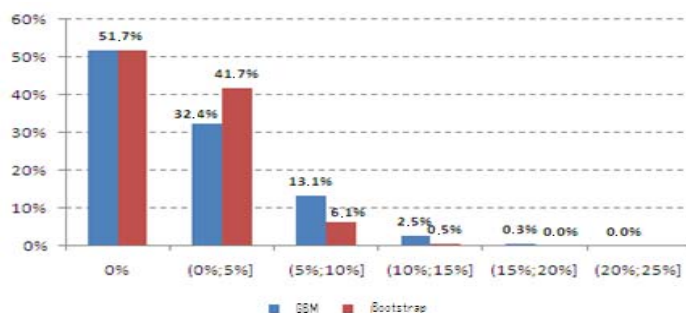
The analysis of this complex financial product was carried out based on two methods: the Geometric Brownian Motion and the bootstrap model.

The results for the parametric Monte Carlo simulation are presented in Chart 6. The probability of no return being paid is 51.7% and the probability of such return (Average Annual Expected Return) being positive and less than 10% is 45.5%. The Average Annual Expected Return for this product is 2%.

The results obtained via bootstrap model point towards an AAER of 1.3%. This is lower than that attained with the Monte Carlo simulation. The probability of nil return is 51.7%. However, the second category of returns (positive and less than 5% AAER) has a probability of greater occurrence than that recorded in the Monte Carlo simulation, leading to a lower overall AAER, the bootstrap model comparing unfavourably with the Monte Carlo outcome.

An alternative to using the GBM stochastic process could be the Variance Gamma stochastic process. However, the kurtosis of the return series of the Index 7 is 1.94 (less than 3), hence making the method unsuitable for the purpose of the analysis envisaged.

Chart 6 – AAER Simulations – CFP F



3.7. CFP G

a) Product Description

This product is capital guaranteed with a 10-year maturity, although early repayment may occur. The return depends on the interbank interest rate. The payment structure includes payment of an annual coupon.

A 5% coupon (T_1) is paid in the first year. Between the 2nd and 9th year, the coupon value depends on the proportion of the number of days the interest rate falls into a given interval. Thus, the coupon value is calculated as follows:

$$\text{Max}\left(0.5\%; \text{Previous Coupon} \times \frac{n}{N}\right)$$

where n is the number of days that the reference rate level is within the relevant interval for the interest payment period and N refers to the total number of days for the interest payment period. The intervals set for each period for interest payment are:

2 nd Year	[2.00%-3.00%]
3 rd Year	[2.00%-3.30%]
4 th Year	[2.00%-3.55%]
5 th Year	[2.00%-3.70%]
6 th Year	[2.00%-3.95%]
7 th Year	[2.00%-4.20%]
8 th Year	[2.00%-4.45%]
9 th Year	[2.00%-4.70%]

However, there is the possibility for early repayment of capital invested. This takes place in the year when the sum of coupons paid is equal to 13.5%. The coupon paid on the redemption date is calculated as follows:

$$T_j = \text{Max} \left\{ 0,5\% ; \text{Min} \left(T_{j-1} \times \frac{n_j}{N_j} ; 13,5\% - \sum_{j=1}^{j-1} \text{Coupon}_j \right) \right\}$$

where T_j is the coupon paid in year j .

In any event, the sum of the coupons paid until repayment will be equal to 13.5%.

Finally, the difference between 13.5% and the sum of the coupons already paid is paid out in the 10th year:

$$T_{10} = 13,5\% - \sum_{j=1}^9 T_j$$

b) Product Evaluation

Interest rates behave differently from shares. While shares display an upward drift over time, interest rates converge towards a long-term equilibrium value. As such, the stochastic process adopted for modelling interest rates should differ from those used in the products analysed so far. The choice of stochastic process fell on the stochastic process proposed by Cox, Ingersoll and Ross, as the nominal interest rates should always be non-negative (which may cripple the use of the stochastic process proposed by Vasicek).

The use of the Cox, Ingersoll and Ross model requires some parameters' estimates, namely, the speed of adjustment towards equilibrium. In order to estimate this parameter, econometric regression was used. Since the parameter found is not statistically meaningful, a zero speed of adjustment was adopted. The non-significance of the coefficient linked with the volatility of interest rate adjustment is also reflected in the non-relevance of the parameter associated with interest rate equilibrium.

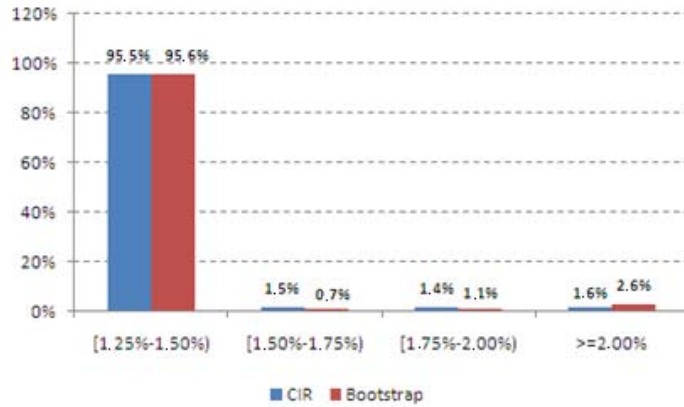
The clearance of payments linked with this product requires the simulation of the interest rate trend on a daily basis, with 1000 simulations⁸, via Monte Carlo method. The Average Annual Expected Return linked with this product is 1.37%. In 95.5% of simulations, the product has a 10-year maturity and pays an annual return (AAER) between 1.25% and 1.50%. Only 4.5% of the simulations showed annual returns greater than 1.5%.

In the case of the bootstrap model, the AAER is 1.38%, which is very close to that obtained from the Monte Carlo simulation. The probability of finding the AAER between 1.25% and 1.50% is 95.6%. The distribution function of payments is also

⁸In this case, only 1,000 simulations were carried out as these needed to be carried out on a daily basis and the computing effort required did not allow for the 25,000 simulations to be maintained as in previous products.

very close to the distribution from the Monte Carlo simulation, which signals the results' robustness.

Chart 7 – AAER Simulations – CFP G



The probability of repayment in the tenth year is above 95% for both methods used. The probability of early repayment is greater in the fifth year when the Bootstrap method is taken into account (1.3% - see Table below).

<i>Redemption (t)</i>	<i>CIR</i>	<i>Bootstrap</i>
5	0.3%	1.3%
6	1.3%	1.3%
7	1.4%	1.1%
8	0.8%	0.4%
9	0.9%	0.8%
10	95.3%	95.1%

Note: Up to the 5th year, the probability of early repayment is zero.

3.8. CFP H

a) Product Description

The maturity of CFP H is November 2010 and its return is linked to positive trends of a basket comprised of four securities of the telecommunications sector (Share 14, Share 15, Share 16 and Share 17, with an equal weighting of 25% for each security).

The product return entails a 100% of the basket's valuation up to a maximum of 20% and a knock-out barrier in the event of the basket's intraday valuation

attaining or exceeding 35%. In case the knock-out barrier is attained, the investor no longer benefits from the basket's appreciation and receives a return of 2.5% at maturity and is no longer exposed to the risk of losing 5% of the capital. The risk is limited to losing 5% of capital at maturity.

The redemption amount (VR) results from the following calculations:

- If $Var.Basket_{max} < 35\%$
VR=Nominal Value x [Max (95%, min (120%, 1+ $Var.Basket_t$))]
- If $Var.Basket_{max} \geq 35\%$
VR= Nominal Value x (1+2.5%)

where,

$$Var.Basket_t = \sum_{i=1}^4 [j_{(i)} \times Var.Share_{(i)}]$$

(Variation mean of four shares of the Basket on each day 't', as against the initial quote)

$$Var.Share_{(i)} = \frac{Share_t}{Share_0} - 1$$

and $j_{(i)}$

(i)	Share_(i)	Weight in Basket (j)
1	Share 14	25%
2	Share 15	25%
3	Share 16	25%
4	Share 17	25%

$Share_t$ = Closing price of *Share i* in each day *t* of the investment's valuation

$Share_0$ = Closing price of *Share i*, on the investment's inception date

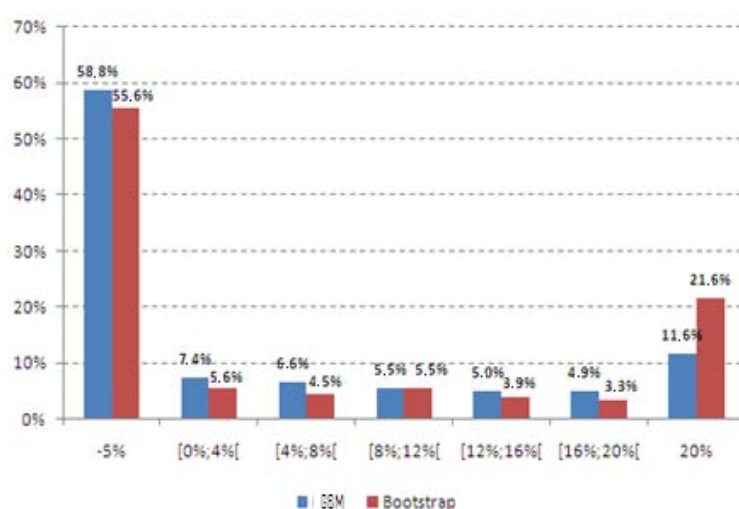
$Var.Basket_{max}$ = Maximum intraday variation of the basket observed between the commencement date and maturity date.

b) Product Evaluation

Simulations were carried out on stock price quotations using two different models: the Geometric Brownian Motion⁹ and the Bootstrap model. The results are displayed in Chart 8. The probability of losing 5% of the capital at maturity is 58.8%; the probability of a positive annual return less than 20% is 30%; and the probability of being paid the maximum bonus of 20% is 11.6%. The AAER for this product is 2.05%. There was never the possibility of a knock-out as envisaged in the prospectus in these simulations.

The Average Annual Expected Return obtained by the Bootstrap method was 3.59%. Only 21.6% of 1,000 simulations resulted in a 20% coupon payment (AAER) and 55.6% of simulations resulted in the loss of 5% of capital invested. Finally, 22.8% of the simulations led to a coupon payment between 0% and 2%.

Chart 8 – AAER Simulations – CFP G



3.9. CFP I

a) Product Description

CFP I has one single interest payment (possible) due in occasion of the bonds' maturity (i.e. at the end of 3rd year of the product lifecycle). The interest payment depends on the performance of three share indices (Index 8, Index 9 and Index 10), and may account for:

- 15% of the initial invested capital (which is equal to an annual nominal rate of return of 5%) if the official closing value of all three indices, observed on

⁹In this case, only 5,000 simulations were carried out as these needed to be carried out on a daily basis and the computing effort required did not allow for the 25,000 simulations to be maintained as in previous products.

the Determination Date, is equal to or greater than the respective level of Barrier 1;

- 0% of the initial capital invested if the official closing value of at least one of the three indices, observed on the Determination Date, is less than the respective level of Barrier 1.

Thus, at maturity, the investor will be repaid 100% of the initial capital invested and receive an interest rate equal to 15% of the initial capital invested if all three indexes' official closing price is equal to or greater than the respective level of Barrier 1. However, the investor will not obtain any return if the official closing price of any of the three indices is lower than the respective level of Barrier 1.

The initial capital invested is at risk, and there is a partial or total loss of invested capital if the official closing price of at least one of the three indices, observed on the Determination Date, is lower than the respective level of Barrier 2. The value of capital loss will be the devaluation of the index with the worst performance compared to the respective level of Barrier 1.

In case the redemption value is less than 100% of the Nominal Value, the bond will record a capital loss that may ultimately be 100% of the Nominal Value.

The values of Barriers 1 and 2 are as follows:

<i>Index i</i>	<i>Barrier 1</i>	<i>Barrier 2</i>
Index 8	3,763.55	2,822.66
Index 9	1,379.93	1,034.95
Index 10	14,012.20	10,509.15

b) Product Evaluation

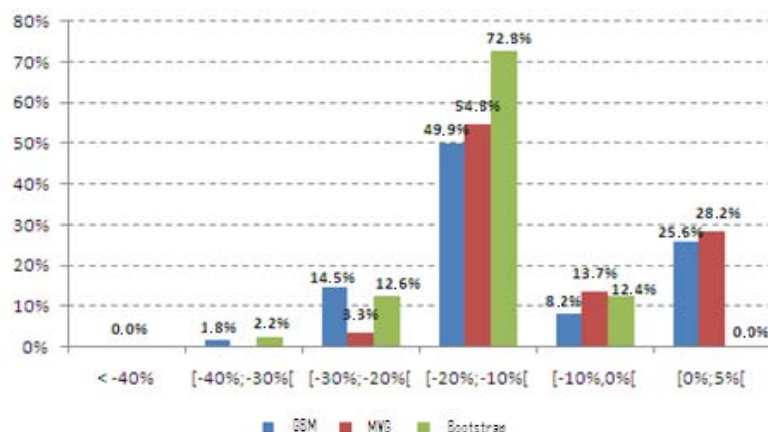
The Geometric Brownian Motion, the Variance Gamma Model and the Bootstrap model were used to simulate the trend of the three share indices mentioned above.

For the GBM method, it appears that the 15%-coupon is paid (in 3 years) only in 8% of the cases, the zero coupon in 21% (0% interest rate) and in 71% of cases there is a probability of a partial capital loss (average loss of 36% of the invested capital). The AAER obtained via this method was -7.79%.

The AAER of the Variance Gamma Model was -5.97%. This simulation shows different characteristics from those of the Geometric Brownian Motion, particularly with regard to the increasingly extreme negative values. There are no losses greater than 35% in this simulation.

The results obtained from the Bootstrap simulation are somewhat different. Thus, the maximum payment coupon of 15% was not ascertained in any of the simulations. The zero-coupon took place in approximately 22% of the simulations and there was capital loss in the remaining 78% cases. The AAER is -11.54%.

Chart 9 – AAER Simulations – CFP I



4. CONCLUSION/SUMMARY

Nine complex financial products, with different characteristics, were selected and analysed. The aim of this work was to test and exemplify the methodologies that can be applied in evaluating this type of products and also to evaluate the quality, consistency and fairness of the forecasts drawn up by financial intermediaries.

The overview table below summarizes the most relevant characteristics of the products analysed and the methodologies applied in each case. In this analysis, products with varying duration (ranging from one to 10 years) and with different types of reference assets (including indices, shares, commodities, interest rates, etc.) were included. The analysis comprises both products with and without capital guarantee.

	<i>Product</i>	<i>Duration</i>	<i>Capital Guaranteed</i>	<i>Return Guaranteed</i>	<i>Early Redemption</i>	<i>Underlying Assets</i>
1	CFP A	3 years	Yes	No	Yes	Shares (Energy)
2	CFP B	5 years	Yes	Yes (1%/year)	No	Shares (Mining)
3	CFP C	5 years	Yes	No	No	Commodities Index and CPI
4	CFP D	3 years	Yes	No	Yes	Share Indices
5	CFP E	5 years	No	No	Yes	Shares
6	CFP F	5 years	Yes	No	No	Commodity Index
7	CFP G	10 years	Yes	Yes (0.5%/year)	Yes	Euribor 12m
8	CFP H	1 year	No	No	Yes	Shares (Telecom)
9	CFP I	3 years	No	No	No	Share Indices

As demonstrated by our analysis, it can be concluded that the evaluation results are not entirely the same for the different methodologies adopted.

Other conclusions can be drawn from the study conducted. Firstly, the overwhelming majority of the complex financial products analysed are quite exotic and, therefore, the distribution of payments associated with different possible scenarios is difficult to grasp by investors, even the most sophisticated investors or those with greater expertise in financial matters. Furthermore, most of these products display a strong return asymmetry for the investor. In the case of capital guaranteed products, for example, there is a strong asymmetry in the probabilities of different scenarios. This can be attributed to a strong probability being concentrated on the scenario of no return being paid or only the minimum return set out in the product technical factsheet being eventually paid. However as a rule, these products marketing advertise the maximum return that the product can attain and neglect the fact that there is a high probability that the scenario entailing the minimum return will may take place. Moreover, it was noted that high return scenarios had considerably low probabilities. In the case of products where there is no guarantee of capital, it becomes even more difficult for investors to understand the respective payment structure, due to the non-linearity of these payments and the number and type of underlying assets usually involved in the determination of the amounts to be paid. The risk of these products is higher precisely because the capital is not guaranteed. This means that the transparency of the respective return scenarios should be intensified in these products. Finally, the study's results indicate that the complex financial products analysed generally have returns lower than the expected returns of traditional term deposits.

APPENDIX – Statistics

A – CFP A

Parameters used in Monte Carlo Simulations

	<i>Share 1</i>	<i>Share 2</i>	<i>Share 3</i>	<i>Share 4</i>	<i>Share 5</i>
Spot Price	3.10	16.20	6.77	26.80	370.00
Interest Rate	1.07%	1.07%	1.07%	1.07%	1.07%
Dividend Yield	5.19%	1.57%	5.00%	5.45%	0.00%
Daily Volatility	1.44%	3.53%	1.54%	2.65%	3.28%
Kurtosis	12.62	9.17	9.43	11.96	7.53
Skewness	0.11	0.35	0.36	-0.36	0.01

Correlation Matrix between the Returns of the Underlying Assets

	<i>Share 1</i>	<i>Share 2</i>	<i>Share 3</i>	<i>Share 4</i>	<i>Share 5</i>
<i>Share 1</i>	1.00	0.34	0.44	0.26	0.24
<i>Share 2</i>	0.34	1.00	0.46	0.32	0.50
<i>Share 3</i>	0.44	0.46	1.00	0.38	0.31
<i>Share 4</i>	0.26	0.32	0.38	1.00	0.21
<i>Share 5</i>	0.24	0.50	0.31	0.21	1.00

Frequency Table

	GBM	Bootstrap
0%	62.1%	54.3%
]0%; 1%]	15.6%	20.1%
]1%; 2%]	6.1%	10.2%
]2%; 3%]	0.3%	2.1%
]3%; 4%]	3.7%	7.4%
]4%; 5%]	1.8%	3.7%
]5%; 6%]	0.1%	2.2%
Average Annual Expected Return	1.10%	0.96%

B – CFP B

Parameters used in Monte Carlo Simulations

	<i>Share 6</i>	<i>Share 7</i>	<i>Share 8</i>	<i>Share 9</i>	<i>Share 10</i>
Spot Price	2093.5	1766	45.83	2750	22.01
Interest Rate	1.07%	1.07%	1.07%	1.07%	1.07%
Dividend Yield	1.95%	3.65%	1%	6.26%	0
Annual Volatility	37.48%	28.84%	28.90%	42.49%	35.41%
Kurtosis	7.67	5.95	9.32	17.19	8.34
Skewness	0.22	0.32	0.69	-0.37	0.04

Correlation Matrix between the Returns of the Underlying Assets

	<i>Share 6</i>	<i>Share 7</i>	<i>Share 8</i>	<i>Share 9</i>	<i>Share 10</i>
<i>Share 6</i>	1				
<i>Share 7</i>	0.83	1			
<i>Share 8</i>	0.22	0.26	1		
<i>Share 9</i>	0.77	0.78	0.23	1	
<i>Share 10</i>	0.49	0.51	0.42	0.49	1

Frequency Table

	GBM	Bootstrap
1%	21.3%	51.6%
1.79%	40.0%	17.5%
2.58%	28.5%	21.5%
3.38%	8.8%	9.3%
4.19%	1.3%	6.7%
5%	0.1%	3.2%
Average Annual Expected Return	1.39%	1.89 %

C – CFP C

Parameters used in Monte Carlo Simulations

	<i>Index 1</i>	<i>Index 2</i>	<i>Index 3</i>
Spot Price	575.63	57.79	222.28
Interest Rate	1.25%	1.25%	1.25%
Dividend Yield	0.00%	0.00%	0.00%
Daily Volatility	2.42%	1.32%	1.60%
Kurtosis	3.247	2.901	2.746
Skewness	-0.344	-0.170	-0.256

Correlation Matrix between the Returns of the Underlying Assets

	<i>Index 1</i>	<i>Index 2</i>	<i>Index 3</i>
<i>Index 1</i>	1.00	0.34	0.32
<i>Index 2</i>	0.34	1.00	0.32
<i>Index 3</i>	0.32	0.32	1.00

Frequency Table

	GBM/MV	Bootstrap
0%	31.8%	42.4%
]0%; 1%]	0.0%	0.0%
]1%; 2%]	15.9%	9.5%
]2%; 3%]	12.1%	6.3%
]3%; 4%]	11.0%	8.0%
]4%; 5%]	0.0%	0.0%
]5%; 6%]	11.2%	9.0%
]6%; 6.6%]	18.0%	24.8%
Average Annual Expected Return	2.72%	2.71%

D – CFP D

Parameters used in Monte Carlo Simulations

	<i>Index 4</i>	<i>Index 5</i>	<i>Index 6</i>
Spot Price	2743.5	1036.19	10034.74
Interest Rate	1.07%	1.07%	1.07%
Dividend Yield	2.95%	1.98%	1%
Annual Volatility	23.16%	20.40%	22.02%
Kurtosis	4.71	8.22	6.95
Skewness	0.16	0.11	-0.11

Correlation Matrix between the Returns of the Underlying Assets

	<i>Index 4</i>	<i>Index 5</i>	<i>Index 6</i>
<i>Index 4</i>	1		
<i>Index 5</i>	0.88	1	
<i>Index 6</i>	0.86	0.92	1

Frequency Table

	GBM	VGM	Bootstrap
0%	49.9%	56.2%	89.0%
5%	33.5%	32.6%	8.0%
4.88%	10.7%	7.1%	1.7%
4.78%	5.9%	4.1%	1.2%
Average Annual Expected Return	2.48%	2.48%	0.59%

E – CFP E

Parameters used in Monte Carlo Simulations

	<i>Share 11</i>	<i>Share 12</i>	<i>Share 13</i>
Spot Price	26.02	8.84	32.68
Interest Rate	1.47%	1.47%	1.47%
Dividend Yield	0.00%	2.13%	3.53%
Daily Volatility	3.69%	2.71%	2.23%
Kurtosis	8.98	6.11	5.16
Skewness	-0.36	-0.30	-0.12

Correlation Matrix between the Returns of the Underlying Assets

	<i>Share 11</i>	<i>Share 12</i>	<i>Share 13</i>
<i>Share 11</i>	1.00	0.30	0.25
<i>Share 12</i>	0.30	1.00	0.28
<i>Share 13</i>	0.25	0.28	1.00

Frequency Table

	GBM	VGM	Bootstrap
]-30%; -20%]	0.7%	0.0%	0.0%
]-20%; -10%]	27.6%	3.2%	22.5%
]-10%; 0%]	39.8%	20.8%	52.6%
]0%; 10%]	11.6%	64.7%	12.2%
]10%; 20%]	7.9%	2.8%	5.2%
]20%; 30%]	0.0%	0.0%	0.0%
]40%; 42.5%]	0.0%	8.5%	7.5%
Average Annual Expected Return	1.03%	5.90%	-1.05%

F – CFP F

Parameters used in Monte Carlo Simulations

	<i>Index 7</i>
Spot Price	494.12
Interest Rate	1.00%
Dividend Yield	0
Daily Volatility	0.98%
Kurtosis	1.944
Skewness	-0.020

Frequency Table

	GBM	Bootstrap
0%	51.7%	51.7%
]0%; 5%]	32.4%	41.7%
]5%; 10%]	13.1%	6.1%
]10%; 15%]	2.5%	0.5%
]15%; 20%]	0.3%	0.0%
]20%; 25%]	0.0%	0.0%
Average Annual Expected Return	2.0%	1.3%

G - CFP G

Frequency Table

	CIR	Bootstrap
[0%-0.25%[0.0%	0.0%
[0.25%%-0.5%[0.0%	0.0%
[0.5%%-0.75%[0.0%	0.0%
[0.75%%-1.00%[0.0%	0.0%
[1.00%-1.25%[0.0%	0.0%
[1.25%-1.50%[95.5%	95.6%
[1.50%-1.75%[1.5%	0.7%
[1.75%-2.00%[1.4%	1.1%
>=2.00%	1.6%	2.6%
Average Annual Expected Return	1.37%	1.38%

H – CFP H

Parameters used in Monte Carlo Simulations

	<i>Share 14</i>	<i>Share 15</i>	<i>Share 16</i>	<i>Share 17</i>
Spot Price	9.595	19.545	8.086	17.39
Interest Rate	1.07%	1.07%	1.07%	1.07%
Dividend Yield	4.17%	3.50%	4%	4.22%
Kurtosis	4.55	4.19	7.02	8.6
Skewness	0.29	0.38	0.36	0.89

Frequency Table

	GBM	Bootstrap
-5%	58.8%	55.6%
[0%;4%[7.4%	5.6%
[4%;8%[6.6%	4.5%
[4%;12%[5.5%	5.5%
[12%;16%[5.0%	3.9%
[16%;20%[4.9%	3.3%
20%	11.6%	21.6%
Average Annual Expected Return	2.05%	3.59 %

I - CFP I

Parameters used in Monte Carlo Simulations

	<i>Index 8</i>	<i>Index 9</i>	<i>Index 10</i>
Spot Price	2706.47	1042.88	9802.95
Interest Rate	1.07%	1.07%	1.07%
Dividend Yield	2.95%	1.98%	1%
Annual Volatility	23.16%	20.40%	22.02%
Kurtosis	4.71	8.22	6.95
Skewness	0.16	0.11	-0.11

Correlation Matrix between the Returns of the Underlying Assets

	<i>Index 8</i>	<i>Index 9</i>	<i>Index 10</i>
<i>Index 8</i>	1		
<i>Index 9</i>	0.88	1	
<i>Index 10</i>	0.86	0.91	1

	GBM	VGM	Bootstrap
< -40%	0.02%	0.00%	0.00%
[-40%; -30%[1.75%	0.00%	2.20%
[-30%; -20%[14.52%	3.32%	12.60%
[-20%; -10%[49.90%	54.78%	72.80%
[-10%; 0%[8.17%	13.69%	12.40%
[0%; 5%[25.64%	28.21%	0.00%
Average Annual Expected Return	-7.79%	-5.97%	-11.54%